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ERROR CONTROL FOR DIGITAL COMMUNICATIONS IN THE TACTICAL ENVIRONMENT

K. Brayer
W. F. Longchamp

DECEMBER 1968

Prepared for

AEROSPACE INSTRUMENTATION PROGRAM OFFICE

ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

D/C

Project 705B
Prepared by
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FOREWORD

This report was prepared by the Communications Techniques Department of The MITRE Corporation, Bedford, Massachusetts, under Contract AF 19(628)-5165. The work was directed by the Ground Instrumentation Engineering Division under the Aerospace Instrumentation Program Office, Air Force Electronic Systems Division, Laurence G. Hanscom Field, Bedford, Massachusetts. Robert E. Forney served as the Air Force Project Engineer for this program, identifiable as ESD (ESSIC) Project 5932, Range Data Transmission.

REVIEW AND APPROVAL

This technical report has been reviewed and is approved.

GEORGE T. GALT, Colonel, USAF
Director of Aerospace Instrumentation
Program Office

ABSTRACT

This paper considers error detection with retransmission and forward error correction for the transmission media used in the tactical environment: i.e., troposcatter and high frequency radio.

The results of extensive error pattern measurements in these two media are reported. In the troposcatter channel, bursts with error densities up to 50% and lasting several seconds, followed by several minutes of error-free transmission, are not uncommon. In HF channels, bursts of similar length with short error-free intervals are frequently found; the error density is only 5%.

A practical approach used in the design of coding systems (which prevent information degradation) incorporates channel testing. The error patterns that are collected are used in simulation of various coding techniques, with the objective of identifying the most promising.

FOREWORD

This paper was prepared for the XVth Symposium of the AGARD Avionics Panel on "Techniques for Data Handling in Tactical Systems," Amsterdam, November 1968.

A survey of the techniques of coding (and some practical performance results) which are applicable to tactical digital data transmission systems is presented. More complete analyses of each of the techniques presented, along with more complete practical performance results, can be found in the references.

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SECTION I

INTRODUCTION

The modernization and updating of command and control for tactical forces is bringing increased use of automated digital data handling in tactical control centers. This automation includes data management by computer and the use of digital inputs such as data from air surveillance radars for both air defense and traffic control purposes. A major advantage of digital input is that it facilitates the machine manipulation of source data to provide rapid updating of situation summaries and status of forces reports and accomplishment of logistical planning and scheduling functions. Consequently, effective digital data transmission among various elements of the tactical forces becomes increasingly necessary. The achievement of accuracy in digital data transmission has been a difficult problem, however, especially in the tactical environment.

All communications systems require some form of authentication or credibility criteria in order that the receiver may have confidence that he has correctly received what the sender transmitted. In manual systems, this authentication function is generally performed by the person receiving the message, and is based on the natural redundancy in the message or on other information that the recipient may possess. In digital data transmission, on the other hand, especially for machine-to-machine communications, the automatic information-processing equipment does not necessarily possess the inherent discriminative capabilities of a human operator. These capabilities can be achieved, however, by the appropriate application of error-control coding techniques. Almost any level of data confidence can be developed in this way.

Likewise, the measures of performance for analog and digital systems differ. In analog transmission, the performance criteria for system evaluation are normally defined in terms of intelligibility, fidelity, or other subjective measures related to the distortion introduced by the transmission links. In digital communications, the most significant performance parameter from either a bit or a message standpoint is that of probability of error. A means to minimize the probability of message error is therefore a prerequisite to effective digital information transfer. An acceptably low probability of error is being achieved in the domestic fixed communications plant. In the tactical environment, however, the communication media employed introduce a host of difficulties in attaining comparable performance.

The communications circuits used in tactical systems are characterized by the large-scale employment of high frequency[1] and troposcatter[2] radio links. Digital data transmission capability is achieved on these links by use of proper modem (modulator/demodulator) equipment on the various communication transmission media. The modems perform the necessary digital-to-analog conversion at the sending terminal and the converse at the receive terminal. Different modems may have different error probabilities as a result of the techniques incorporated in the modem design. Thus, in defining the digital communication channel, the modems must be considered as an integral part of the channel, as illustrated in Figure 1. The behavior of this channel, then, can be completely described by the occurrence or non-occurrence[3, 4] of errors.

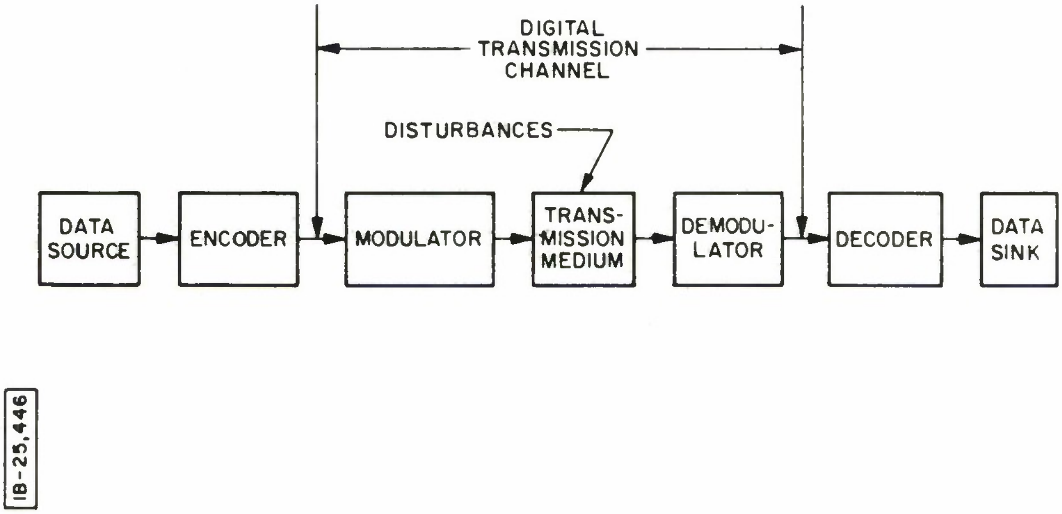

Figure 1. Error Control on Digital Transmission Channel

Investigations [3, 4] of the manner in which errors occur on real channels have shown that errors generally occur in an extremely non-random fashion. Consequently, it has not been possible to describe channel error behavior by a simple stochastic model. Several investigators have attempted to develop channel models which represent the observed error behavior, but in general those models [5, 6, 7, 8] which are simple enough to be analytically tractable do not provide sufficient complexity to adequately describe the error patterns or to satisfactorily predict the performance of error-control techniques on real channels. Thus the evaluation of error-control techniques is accomplished through measurements of real channel-error patterns and computer simulation of codes.

Developments in the field of digital coding theory have provided powerful tools whereby techniques can be devised for detecting and/or correcting errors introduced in the digital data transmission process. These codes introduce a controlled redundancy into the transmitted messages in such a manner that the decoder can determine whether or not an error exists within a received message. With an error-correcting code the position of the error can be determined, and consequently the error can be corrected. An extensive variety [9] of codes have been devised which provide varying degrees of detection and correction capability associated with several levels of implementation complexity.

SECTION II

ERROR CONTROL TECHNIQUES

The techniques of error control can be divided into two classes: error detection and error correction (see Figure 2). Error detection can be achieved by either adding redundancy or using redundancy that may already be available. This redundancy then permits the detection of errors that occur in transmission. Error correction can be performed if the redundancy is such that the portion of the information which is in error can be located. Two of the possible forms of error control are:

1. Error detection with retransmission

2. Forward error correction (no retransmission)

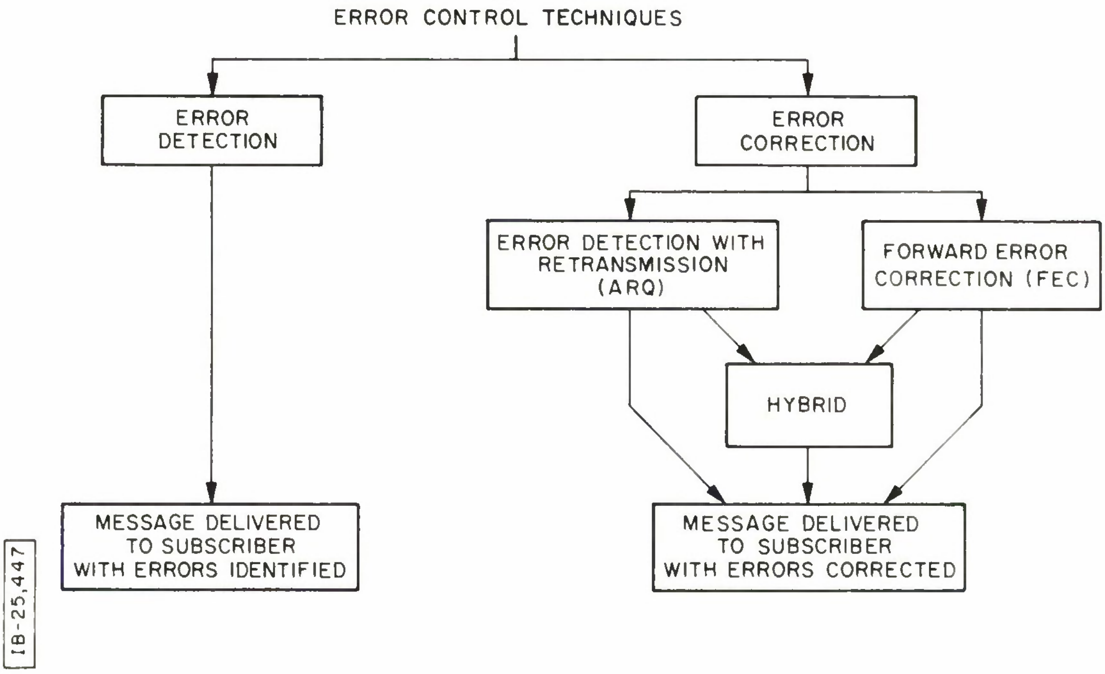

Figure 2. Classes of Error Control Techniques

ERROR DETECTION WITH RETRANSMISSION

In this technique, message segments with errors are detected and the transmitter is asked to repeat those segments (see Figure 3). For example, using the source information in each message segment, a check number which is a function of the information is calculated. The check number is appended to each message segment. At the receiver the information in each message segment is used to calculate the same function, and the two numbers are compared. If the check numbers disagree, it is assumed that the message segment (including the number) is in error, and a repeat is requested. Unfortunately, all possible patterns of errors may not be detected.

Repeat systems generally require the following:

(a) Full duplex communication — one channel for information and a return channel for repeat requests.

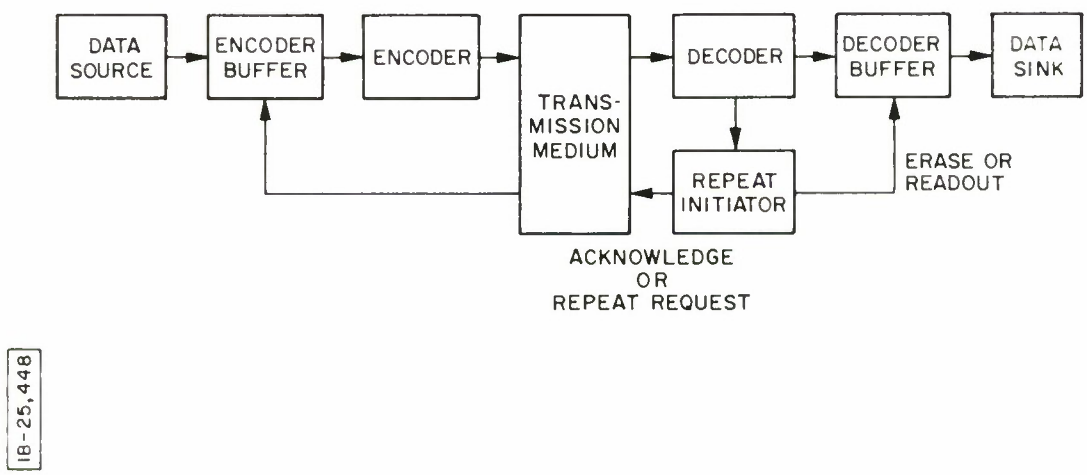

Figure 3. Retransmission System

(b) Identification words on each message segment — in order that proper message accounting can be maintained.

(c) Buffer storage — for holding message segments until correctly received.

Some of the disadvantages of the retransmission technique are:

(a) Lock-up — if the error rate in the channel becomes high, the system might continually be repeating the same segment.

(b) Variable delay — different segments may be repeated different numbers of times.

It is thus evident that this approach cannot be used if throughput must be maintained at all times (lock-up not permitted), or if data must be received in something near real time (unrestricted or variable delay not permitted), or if only a one-way transmission link is permissible (full duplex operation not possible). In Section IV the performance of a common error detection technique used in conjunction with retransmission techniques will be described.

FORWARD ERROR CORRECTION

In forward error correction,[9] enough redundancy of a specific mathematical form is added so that each bit in error is located and the nature of the error is found within the capability of the code. All codes will have some uncorrectable messages which lead to error generation. A forward-error-control system (Figure 4) does not use retransmission; it therefore works with a simplex channel, cannot lock-up, and has fixed delay which is a function of the decoding time for the code selected. It does have the disadvantage that a detailed knowledge of the channel error patterns is required

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Figure 4. Forward Error Correction System

in order to select codes; otherwise there may be a high probability of uncorrected or generated errors and the decoding delay may be excessive. In general, all design parameters are interrelated, and, having chosen one, the design is constrained with regard to the others.

HYBRID ERROR CONTROL

The hybrid error-control[10] technique employs both forward error correction and error detection with retransmission. (Figure 2.)

As a combination of two coding procedures, hybrid error control exhibits a number of advantages. In general, the forward-error-correction feature of the hybrid technique reduces the number of retransmissions required, while the retransmission feature ensures that blocks which cannot be handled by forward error correction are retransmitted. Specifically, in a random-error environment, the forward-error-correcting code will provide error correction within the code block, and the retransmission feature will eliminate any residual errors. In a high-density burst-error[10] environment the retransmission will be used for error correction, while the forward error control eliminates the errors in the intervals between bursts. If the burst-error density increases to an extremely high level and the bursts get excessively long, or if random errors occur with a bit error rate higher than a threshold (percentage of correctable error bits) derived from the code, then the system will lock-up. However, it is possible to encode in a way that avoids this.[10]

The hybrid error-control system operates according to the following transmission procedure. Source information is encoded with an error-detecting code and then encoded with an error-correcting code. The information is now transmitted in blocks to a receiver site where the error correction is accomplished. After error correction, a final detection is made for residual errors. If there are no errors, the data are transferred to the data sink. If there are errors, the block identification number is recorded for future use by the retransmission system and the block is rejected. During the finite time that it takes to decode one block of information, the next block can be transmitted. If transmission were interrupted for retransmission of rejected blocks, the finite delay would cause a system slowdown. Therefore, retransmission requests transmitted back to the data source initiate block recoding at the conclusion of the initial transmission. The only case in which there can be delay time between blocks is when the block to be retransmitted is the last block.

The technique just described can be used for transmission of non-real-time information. If near-real-time transfer is required, a request for retransmission can be initiated immediately and the rejected block in question will be regenerated and retransmitted (following transmission of the block in transit when the request is made).

The performance of a hybrid system (with its error-correction function inhibited) as a function of block length and delay is presented in Figure 5. It is demonstrated in Equation (1) of the Appendix that for a given value of block length, as the probability of error approaches zero, the efficiency of the system approaches the code rate (the upper limit of efficiency[10]). Similarly, for a given probability of error, the efficiency will approach zero as the block length is increased. It is evident from Figure 5 that for any probability of error and delay there is an optimum block length. (The curves presented are for a theoretical channel. See derivation in Appendix. Practical curves can be found in Brayer, Reference 10.)

METHOD OF SELECTING ERROR-CONTROL TECHNIQUES

For the two classes of error control considered in this paper there are numerous codes and message structures that can be implemented. Within the class of error detection for retransmission systems, the error detection can be implemented using cyclic codes, block codes, horizontal and/or vertical parity checks, arithmetic codes, etc. Message segments can be transmitted with spaces between them allowing processing time, or with no spaces and all processing at the end of the message. Repeats can be made immediately or at some later point in time. The return link may be coded to reduce the chance of errors in the repeat requests.

Theoretical Approach

One approach to technique selection common in communication systems is that of theoretical analysis. An example of this, from modulation systems, is the selection of a message format which, when used in conjunction with the mathematical model of the communications channel, allows the analyst to calculate the best detection strategy. For error control, this technique of selection

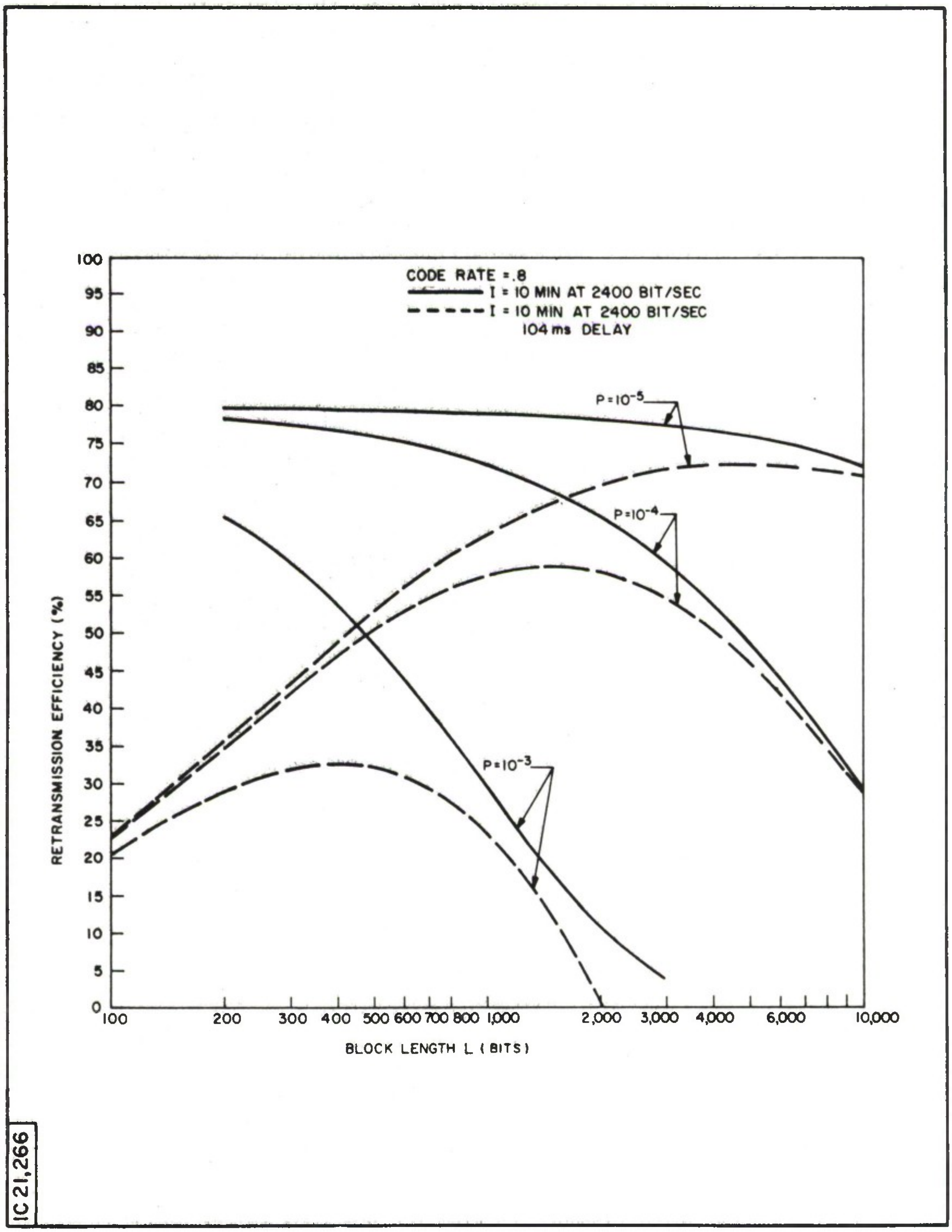

Figure 5. Efficiency vs. Block Length for a 10-minute Message (I) As a Function of Bit Error Rate (P) in a Binary Symmetric Channel

involves problems. While there are numerous models of channels available for such calculations, none, as stated previously, has been shown to adequately represent a real channel. Thus, the theoretical calculations of expected performance of a coding technique that can be produced have little practical application. In the absence of a theoretical approach a practical approach can be adopted.

Practical Approach

A practical approach which has proven useful overcomes the theoretical limitations by (1) collecting (through testing) a representative sample of error patterns from the channel on which coding is to be applied, and (2) evaluating various coding techniques by computer simulations, with the objective of identifying the most promising technique.

SECTION III

REAL CHANNEL DATA DESCRIPTION

This section describes the patterns of errors that have been measured on actual high-frequency and troposcatter radio channels. Although these were long-haul circuits, the resulting data appear similar to tactical data previously reported.[1,2]

GENERAL DESCRIPTION OF DATA

In October of 1965, field tests were conducted on HF circuits between Antigua and Ascension Island to record HF digital error patterns.[3] Tests were conducted for 10-minute runs at 1200 and 2400 bits/sec using a Kineplex TE-216, phase-shift-keyed, 16-tone modem. The procedure was to transmit a 52-bit test message from Antigua to Ascension and back to Antigua on a looped basis. At Antigua the received message was compared with a suitably delayed replica of the 52-bit test message, and bits which were not in agreement were declared in error. The results were recorded on magnetic tape and converted to IBM 7-track format magnetic tape for use in the IBM 7030 computer.

The troposcatter data were collected in the spring of 1966 on a link having three types of transmission media: wireline, microwave, and troposcatter. The troposcatter[4] is multiple hop, and the wireline consists of the interconnection of numerous leased telephone wirelines. The dominant sub-path is a troposcatter hop of 583 miles. During this test Sebit 24B vestigial sideband AM modems were used at 1200 and 2400 bits/sec in a 4 kHz channel FM multiplexed with other channels for transmission on the dominant subpath. The test procedure was the same as for the HF tests except that the transmission was on a one-way basis and runs were 90 minutes long.

Analysis of the HF data indicates that errors occur primarily in bursts. A typical burst is from 2000 to 4000 bits long with an average error density of approximately 5 percent. Bursts are separated by intervals that have a low error density and vary in length from one-tenth to as much as a million times the length of the burst they follow. Additionally, periodic errors of 32-bit spacing occur in the output serial data stream as a result of tone interference on individual tones (16 tones, 2 bits/tone/baud) of the TE-216 modem. The average error rate is 5.47×10^{-3}.

The data collected on the multiple-hop tropo path[4] consist of bursts that vary in length from a few hundred bits to tens of thousands of bits where for the most part the density of errors is as much as 50 percent. The intervals between bursts are error-free and vary from one-tenth the length of the burst they follow to one million times the length. The average error rate is 3.95×10^{-4}.

STATISTICAL DATA ANALYSIS

The key statistics of channel data are those of consecutive errors, error-free gaps, message error rates, and bursts.

The statistics of the combinations of modems and transmission media for consecutive errors and gaps are presented in Figures 6 to 9. Assuming independent random errors the theoretical equations for consecutive errors and gap distribution are:

$$P\,\{n \text{ consecutive errors}\} = P\{e^n\, c \,|\, c\} = \sum_{k=o}^{n} p^k\,(1-p)$$

$$P\,\{n \text{ bit gap}\} = P\{c^n\, e \,|\, e\} = \sum_{k=o}^{n} p(1-p)^k$$

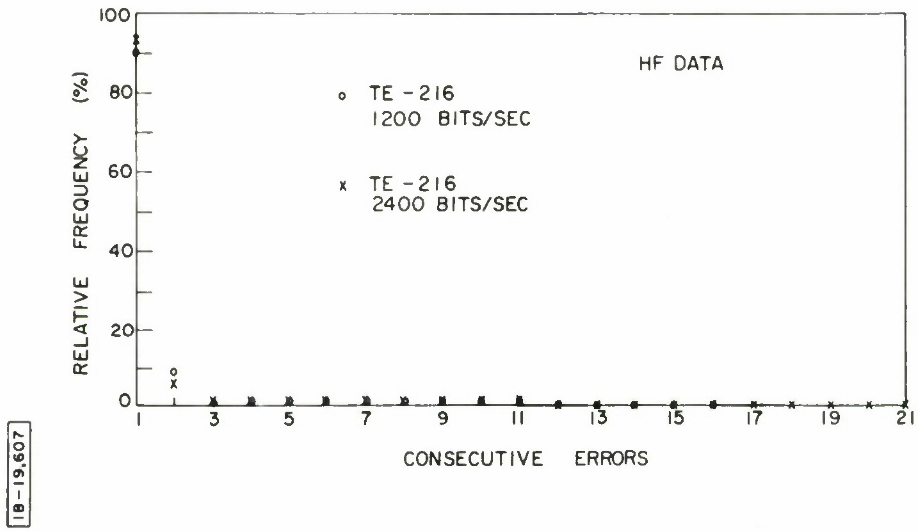

Figure 6. Frequency of Consecutive Error Occurrence (HF).

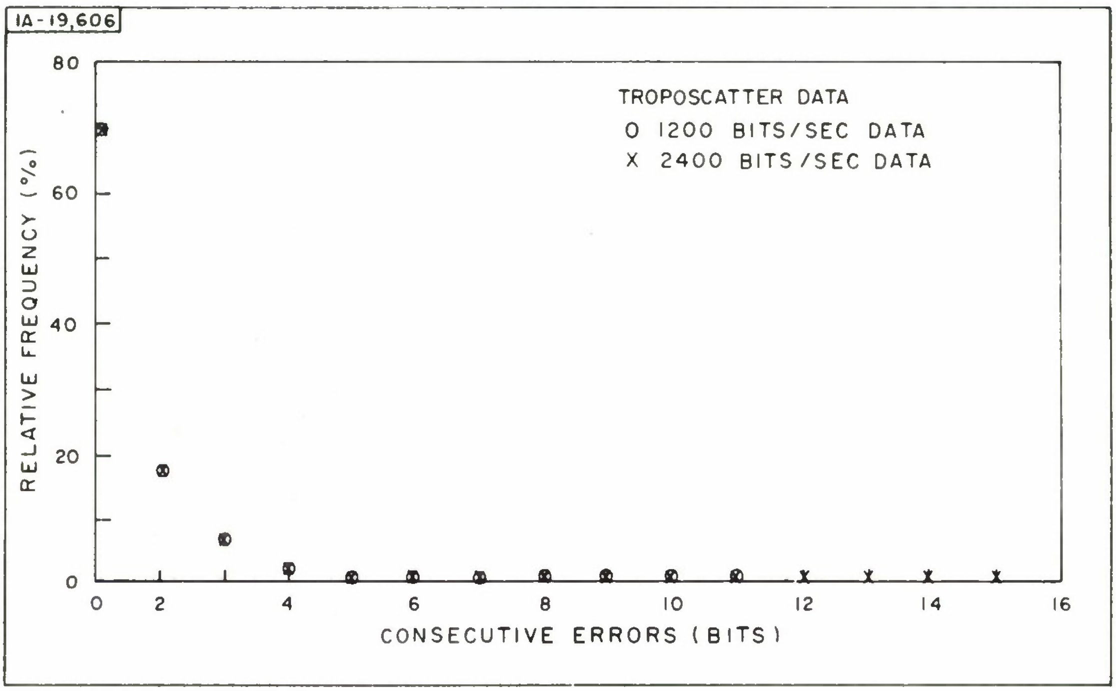

Figure 7. Frequency of Consecutive Error Occurrence (Troposcatter)

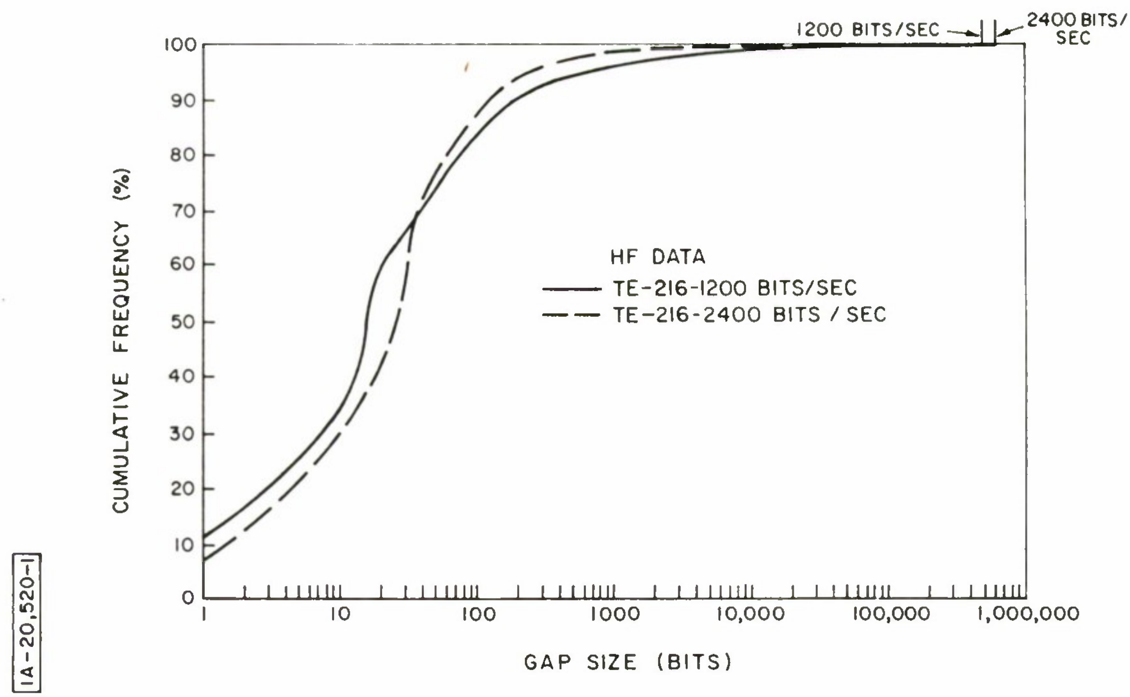

Figure 8. Distribution of Gaps between Errors (HF)

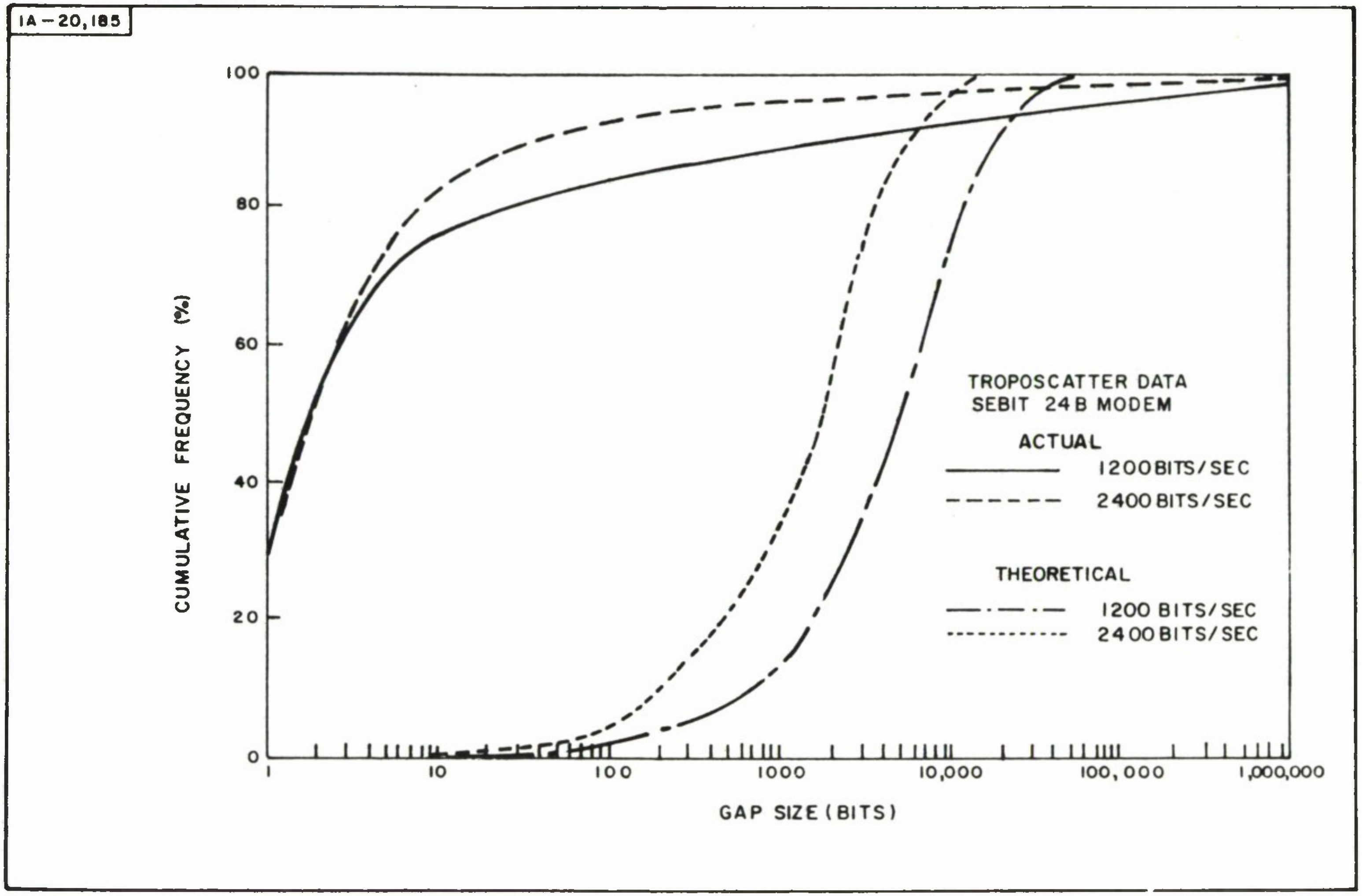

Figure 9. Distribution of Gaps between Errors (Troposcatter)

where

$P\{\cdot\}$	=	cumulative probability function
e	=	an error bit
c	=	a correct bit
n	=	number of consecutive bits
p	=	binary inversion probability (assumed equal to average error rate for practical evaluations)

The theoretical relationship indicates, for the occurrence of consecutive errors, that over 98 percent of the errors should occur as single errors. In practice, the range is from 70 to 94 percent. More than the theoretically expected numbers of multiple errors are occurring, which indicates that the errors are tending to cluster. It can also be seen from the distribution of gaps that there are inordinately high frequencies of short gaps in measured data as opposed to the theoretical distribution (Figure 9). Thus, not only are the errors clustered but the clusters are together, indicating the occurrence of bursts of errors as opposed to the occurrence of random errors. In the case of the TE-216 modem, there is also an occurrence of periodic errors. This is indicated by the high relative frequencies of gaps (16 and 32). It is thought that this is due to the fact that transmission was on parallel tones in a highly congested communications traffic pattern where interference occurred on some tones but not others. After the parallel-to-serial conversion operation which follows detection in the modem, these errors occur periodically. The values of periodicity are the modem frame lengths in bits, as would be indicated by the explanation of tone interference.

The theoretical message error rates (Figure 10) as a function of message size are derived from the relation, which holds for independent errors,

$$p_m = 1 - (1 - p_e)^m$$

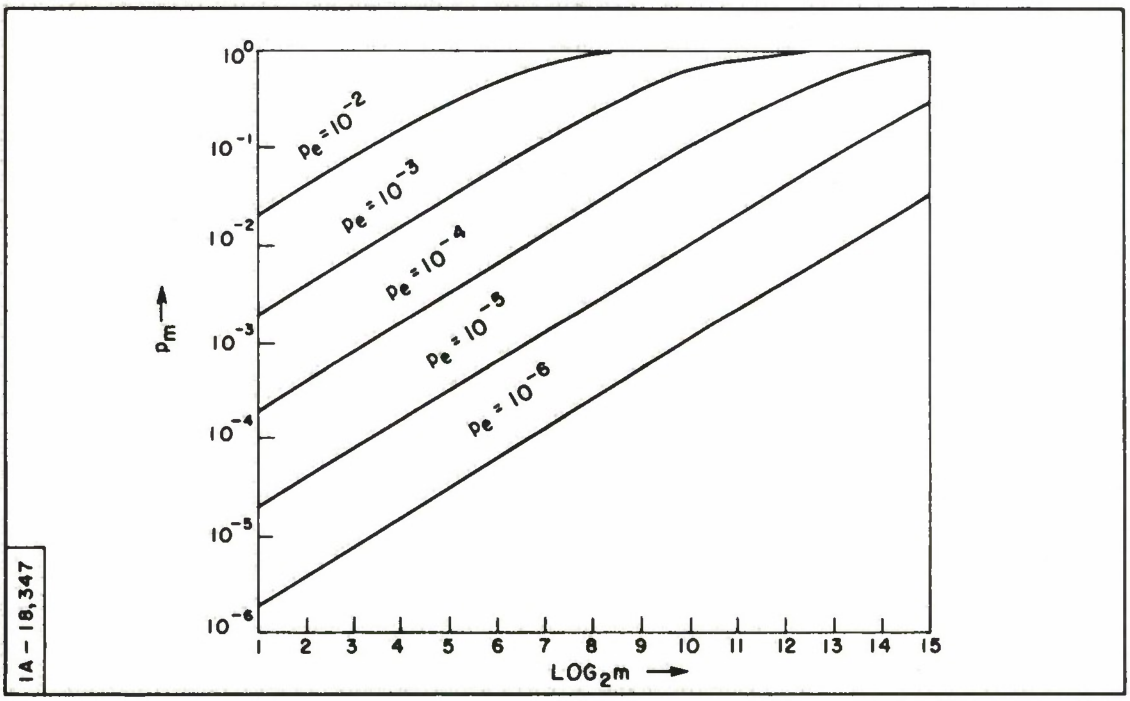

Figure 10. Error Probability of m-Bit Messages p_m Versus m and Error Probability p_e

where

m = message length (bits)

p_e = probability of bit error

p_m = probability of message error

The actual message error rates can be used in the design of block retransmission systems, since they allow the selection of a message size such that the probability of message error is not excessive. They will be displayed later in this paper (Figure 11).

The burst distributions will not be dealt with in this paper. Extensive details are available elsewhere [3,4].

SECTION IV

DESIGN AND PERFORMANCE OF ERROR-CONTROL EQUIPMENT

In recent years numerous commercial error-control devices have come onto the market. The devices are available for forward error control and retransmission error control (hybrid devices are not yet available, although they show great promise for the future).[10]

Forward error-control devices are, in general, designed for the user who is restricted to simplex communication only. The code most frequently used is a 50 percent redundancy code, since the available standard data rates of modulation equipment are restricted to be of the form 75×2^m $(m = 1, 2, \ldots)$. Most devices give the user a choice in the amount of delay introduced into his communication system.

Retransmission error control requires only an error-detection code, and is employed by users who have full duplex communication capability. The primary present consideration in the design of the code is simplicity. Thus, many present systems employ horizontal and/or vertical parity check codes.[11]

In this section the performance of a common forward error-control system and the horizontal/vertical parity check detection systems will be evaluated on the high-frequency and troposcatter channels previously described. The two approaches to error control will be compared, and the relative merits of each will be detailed.

FORWARD ERROR CONTROL PERFORMANCE

Previous papers[12, 13] have described the performance of the modified Golay code. This code generates 12 redundancy bits for 12 source information bits according to a mathematical rule, and will correct 3 errors in the total of 24 bits and detect a fourth error. The performance of the code was simulated using the IBM 7030 computer against both types of data. The gross results of the performance of the code are presented in Table I. It is evident from the table that in the high-frequency channel with a total delay time of only 24 bit-times, an order of magnitude of improvement in bit error rate was achieved.

Table I

Performance of Modified Golay Code

	Troposcatter Channel	High-Frequency Channel
Number of Bits in Test Sample	5.26×10^{8}	4.89×10^{7}
Average Bit Error Rate	3.95×10^{-4}	5.47×10^{-3}
Average Bit Error Rate After Decoding	3.14×10^{-4}	4.05×10^{-4}

While bit error rate improvement is interesting, a practical communications systems has as a more important parameter the message error rate. A message is defined by the user as his basic unit of information transfer and consists of a number of bits.

In Figure 11 the message error rate as a function of message length is presented both before and after decoding. It is demonstrated in this figure that for certain message lengths the decoded message error rate actually exceeds that of the channel. This is not unexpected, since a message of

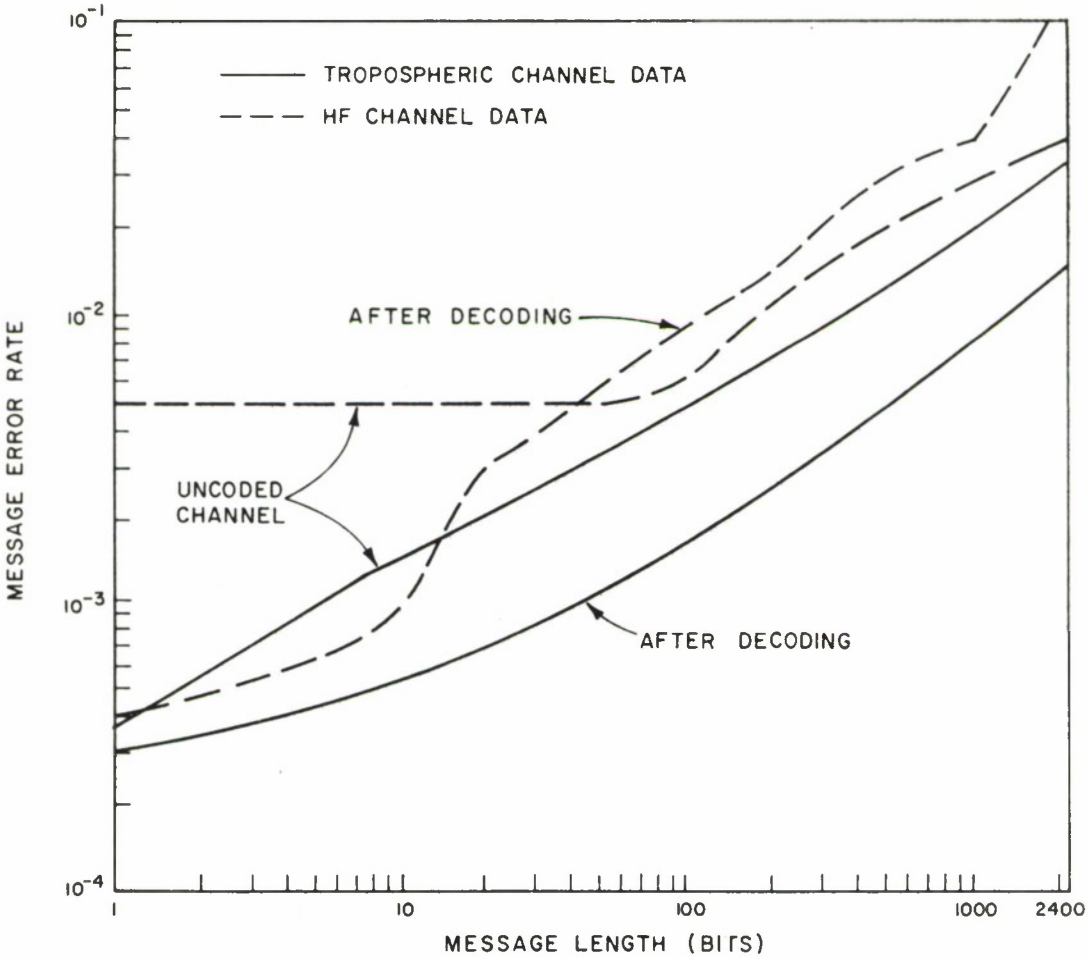

Figure 11. The Effect of Coding on Message Error Rate

length m in the channel becomes one of length m/2 after decoding when the code rate is 1/2, and therefore, failure to correct all message errors will lead to a lateral translation of the curve.*

The user of this coding system defines overall performance by three numbers. The first is the percent redundancy introduced by the code. The greater this number, the lower is the capacity of the channel to transfer information. The second is delay. The greater this number, the more the information falls behind real time. The third is the ratio of channel message error rate to decoded message error rate. This number (for the message length of interest to the user) shows the improvement due to the code. The objective of code design is to simultaneously minimize the first two and maximize the third.

PERFORMANCE OF TWO-DIMENSIONAL PARITY CHECK ERROR DETECTION

As a part of a retransmission error control system an inexpensive but efficient scheme for detecting errors is necessary. The commonly used technique is parity check verification. In this scheme the message, constructed of binary digits, is divided into segments called words. To each word is appended a bit called the horizontal (or row) check bit. The bit is calculated by finding the binary sum, with no carry, of the bits in the word. This can easily be done using a flip-flop that is triggered for each "one." The content of the flip-flop after the word is processed is the parity check bit which is appended to the word. The flip-flop is then reset and the next

*For example, if the message error rate is 0.1 for m-bit channel messages and all messages are not corrected, it will still be 0.1 for decoded data, of length m/2.

word is processed. Thus, independent of the number of rows in the message, only one flip-flop is required. The vertical parity is obtained by appending to the message a word which has, for its check bits, parity on corresponding positions in the previous words (i.e., the first bit is constructed from the first bits of all previous words in the message). As many flip-flops as there are information bits/word are required.

The message so constructed is now transmitted through the channel where errors cause some of the bits to be inverted. The decoding procedure is to calculate a set of parity bits from the received information portion of the message and compare these bits with the corresponding received parity bits. If there is a disagreement, the message (block) has been detected to be in error. This scheme detects changes in parity and as such can only detect the occurrence of an odd number of errors. An even number of errors would not change the parity and would go as undetected errors. In Table II the performance of this scheme for a 9(bits/word)-column and 8(words/message)-row block on the tropospheric data is presented.

From Table II it is found that one order-of-magnitude improvement in block error rate is obtained using row detection only. The cost is one flip-flop each for encoder and decoder. Using column detection only, at a cost of eight flip-flops each, two orders-of-magnitude improvement are possible, and using both row and column detection, nine flip-flops each for encoder and decoder, four orders-of-magnitude improvement is obtained. The example of Table II represents the transmission of a block composed of seven 8-bit computer bytes with the additional parity to create a 9-column, 8-row message. The results, of course, hold for any user of a like-sized message.

In Figure 12 the results for 8-bit bytes, using messages of from 5 to 205 words, are presented for both the tropospheric and high-frequency data. In the case of the tropospheric data the results follow those in the table.

Additionally, for numerous block sizes the undetected block error rate is zero. Those cases where it is non-zero are caused by four patterns of errors of 72-bit length which, depending on the way they occurred within the block structure, were not always detectable. In the case of the high-frequency data the bursts are of low density, and most 9-bit words (rows) have only one error. Thus, row parity is favored over column parity in performance. In both channel media the undetected error rates do not show bottoming. It is concluded that the performance results obtained are limited solely by the size of the data sample used. Therefore, from these results it can only be stated that the undetected error rates are less than 1 out of 10^7 messages.

Table II

Performance of Horizontal/Vertical Parity Checks

Block Size	9 columns x 8 rows = 72 bits
Total Blocks	7,309,865
Blocks in Error	27,181
Blocks Detectable Using Row Parity Only	25,322
Blocks Detectable Using Column Parity Only	26,984
Blocks Detectable Using Two-Dimensional Parity	27,177
Original Block Error Rate	.0037
Undetected Block Error Rate After Row Detection	.00025
Undetected Block Error Rate After Column Detection	.000026
Undetected Block Error Rate After Two-Dimensional Detection	.00000055

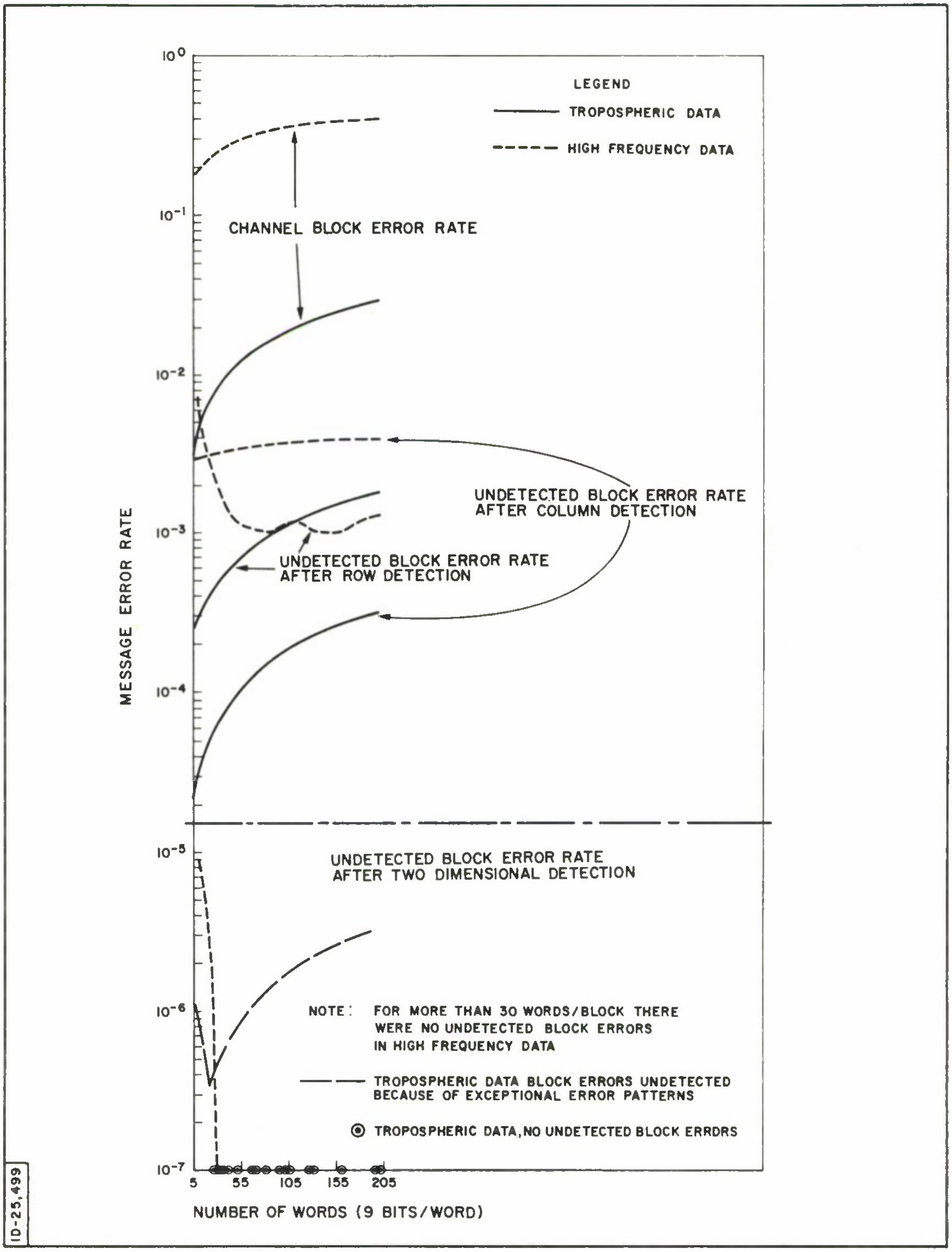

Figure 12. Performance of Two-Dimensional Parity Check Error Detection (Computer Bytes)

SECTION V
CONCLUDING REMARKS

In a tactical environment employing automated digital data handling, the attainment of accurate data communication requires the use of techniques for detecting and correcting errors. Two basic error-control techniques, error detection with retransmission and forward error correction, have been described. Application of these techniques to digital data transmission have been shown to offer substantial improvement in the accuracy of data transmission when properly implemented, especially for the types of communication media that are used in tactical operations.

In order to select and effectively apply the error-detecting and/or correcting codes, the precise nature of the error patterns to be encountered must be known. Tests conducted to collect error patterns for high frequency and troposcatter radio, the communication media that are commonly employed in the tactical environment, show a substantial degree of error clustering. Since these error patterns are a function of both the communications media and the modem equipment to be used, an error-pattern measurement program may be necessary prior to establishing equipment design criteria. The development of design criteria based on computer simulation of code performance will then ensure a device yielding satisfactory performance.

The objective in the design of forward error-control equipment is to achieve the maximum reduction in error rate with the minimum hardware complexity. In general, the longer the code the greater the error-correcting capability but the more complex and expensive the implementation. In a retransmission system, the design objective is to establish the retransmission block length for optimum information throughput. This requires either that the number of retransmissions be minimized, or that the block length be designed to minimize the probability of one or more errors in a large percentage of the blocks. The error detecting codes used in retransmission systems do not have the complex implementation problems of the forward error-correcting codes.

APPENDIX

DERIVATIONS OF THEORETICAL PERFORMANCE

RETRANSMISSION ERROR CONTROL IN A BINARY SYMMETRIC CHANNEL

A binary symmetric channel is one in which a binary digit is inverted with probability p independent of that which occurs to other digits. The probability on a non-inversion is $1 - p$, or q. If it is assumed that all errors are detected and retransmissions occur for every detection, it is found that

$$P\{\text{retransmission}\} = \frac{\text{messages retransmitted}}{\text{total messages}}$$

and
$$I_R = \frac{I \cdot P\{\text{retransmission}\}}{1 - P\{\text{retransmission}\}} = \frac{I \cdot P\{\text{detection}\}}{1 - P\{\text{detection}\}}$$

where I is the information to be transmitted (bits)

I_R is the retransmitted information (bits).

Since all errors are detected,

$$P\{\text{detection}\} = P\{\text{occurrence of a block error}\} = 1-(1-p)^L$$

for block length L.

Thus
$$I_R = \frac{[1-(1-p)^L]\, I}{(1-p)^L} .$$

The efficiency (E) of transmission, which is given by the ratio of bits of information to total bits transmitted, is given by

$$E = \frac{(1 - p)^L}{1 + R} \tag{1}$$

where R is the code rate. These results are presented in Figure 5, along with results for 104 ms delay.

FORWARD ERROR CORRECTION IN A BINARY SYMMETRIC CHANNEL

Using forward error correction only in the independent error environment, the following relationship is found:

$$P\{\text{correction}\} \approx P\{\text{not more than } e \text{ digit errors in a block}\}$$

$$= \sum_{i=o}^{e} \binom{n}{i} p^i (q)^{n-i} \tag{2}$$

where n is the length of the code (bits)

e is the number of correctable (bits).

For a message made up of m binary digits it is found that the message

$$\text{improvement factor} = \frac{\text{expected number of message errors input}}{\text{number of message errors output}}.$$

$$= \frac{p}{p - \frac{1}{n} \sum_{i=o}^{e} i \binom{n}{i} p^i q^{n-i}} \tag{3}$$

and

$$p = p_m = 1 - (1 - p_e)^m.$$

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13. ABSTRACT

This paper considers error detection with retransmission and forward error correction for the transmission media used in the tactical environment: i.e., troposcatter and high frequency radio.

The results of extensive error pattern measurements in these two media are reported. In the troposcatter channel, bursts with error densities up to 50% and lasting several seconds, followed by several minutes of error-free transmission, are not uncommon. In HF channels, bursts of similar length with short error-free intervals are frequently found; the error density is only 5%.

A practical approach used in the design of coding systems (which prevent information degradation) incorporates channel testing. The error patterns that are collected are used in simulation of various coding techniques, with the objective of identifying the most promising.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
SYSTEMS AND MECHANISMS						
Data Transmission Systems						
Multi-Channel Radio Systems						
Voice Communication Systems						
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INFORMATION THEORY						
Coding						
MATHEMATICS						
Statistical Analysis						
Statistical Distributions						
Statistical Data						